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SOME NEW DISTANCE-4 CONSTANT WEIGHT CODES

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Abstract. Improved binary constant weight codes with minimum distance 4 are constructed. A table with bounds on the chromatic number of small Johnson graphs is given.

1. Introduction

A binary constant weight code of word length n, weight w, and minimum distance d is a collection of (0,1)-vectors of length n, all having w ones and $n-w$ zeros, such that any two of these vectors differ in at least d places. The maximum size of such a code is denoted by $A(n,d,w)$. In this note we give improved lower bounds for $A(n,d,w)$ for $d=4$ and smallish n.

The standard reference for constructions of binary constant weight codes of length at most 28 is [3]. One of the constructions discussed there depends on the existence of partitions of all words of a given length and weight into codes with minimum distance at least 4 (that is, on proper colorings of the Johnson graph). Such partitions are typically found using some form of heuristic search, and today it is easy to improve on the results of [3]. For example, [3] says that $A(22,4,11) \geq 39688$, while we find $A(22,4,11) \geq 40624$ here. Earlier improvements have been given in [9] and [10]. The bounds here improve all but one of the bounds from [9] and all from [10]. For example, [3] gives $A(26,4,13) \geq 424868$, [10] gives $A(26,4,13) \geq 425950$, and we find $A(26,4,13) \geq 431672$.

Motivated by an application to frequency hopping lists in radio networks, the authors of [18] extended the tables of constant weight codes to word length 63 for some values of w and d. For the case of $d=4$ they give bounds on $A(n,4,5)$. Table 3 below gives improvements.

In Section 2 we present five direct constructions, by using some groups, showing that $A(15,4,6) \geq 399$, $A(16,4,5) \geq 322$, $A(16,4,6) \geq 616$, $A(18,4,5) \geq 544$, and $A(21,4,5) \geq 1113$. In Section 3 we describe the partitioning methods and some of its variants. A table of new bounds obtained by partitioning is given and the parameters of the partitions used are presented. Finally, Section 4 contains a discussion of the chromatic number of Johnson graphs and determines this number in a few new cases.

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2. Direct constructions

2.1. $A(15,4,6) \geq 399$. We show that $A(15,4,6) \geq 399$ by using the group of order 21 (that permutes the 15 coordinate positions, numbered right-to-left 0–14) that fixes position 14, and acts on positions 0–13 with the two generators (0,2,1,4,5,3,6)(7,9,8,11,12,10,13) and (0,2,4)(1,3,5)(7,9,11)(8,10,12). The 23 base blocks are:

000000010011111	000001111100100	100001110000101
100000011011001	000001110110001	000101111000010
000000111011010	100000110101100	100101010001001
100000010111010	000011010001101	100011010010001
100000010100111	000001110101010	100001111010000
000001110010110	000101010010101	000111110010000
000101010001110	000011010011010	100011110000010
000001111001001	000011011110000	

2.2. $A(16,4,5) \geq 322$. We show that $A(16,4,5) \geq 322$ by using the group of order 21 that fixes the first two coordinate positions, and acts on positions 0–13 with the two generators (0,2,1,4,5,3,6)(7,9,8,11,12,10,13) and (0,2,4)(1,3,5)(7,9,11)(8,10,12). The 20 base blocks are:

0000000011011001	0000101010000011	0000001111001000
0000000010111010	0000001110100010	0100101011000000
0000000010100111	1000000110000101	1100000111000000
0100000010011100	0000011010011000	1000011010000010
1000000010010110	0000001110010001	0100011110000000
1100000000001101	1000000110001010	1000111100000000
0100000110000011	0100000110110000	

2.3. $A(16,4,6) \geq 616$. We show that $A(16,4,6) \geq 616$ by using a group of order 32 isomorphic to the direct product $C_2 \times D_{16}$, generated by the three permutations

(0,1,2,3,4,5,6,7)(8,9,10,11,12,13,14,15),
(0,1)(2,7)(3,6)(4,5)(8,9)(10,15)(11,14)(12,13),
(0,8)(1,9)(2,10)(3,11)(4,12)(5,13)(6,14)(7,15).

The 27 base blocks are:

0000000111000111	0000011101001010	0000101101010100
0000000100101111	0000011101110000	0000101101101000
0000000101110011	0000100110110001	0001001110100010
0000000110111010	0000100110010110	0001001101010010
0000010101011100	0000001110110100	0001001101100100
0001000100111001	0000101100010011	0001001111001000
0000011100000111	0000101100001101	0001000101010101
0000011100011001	0000101110001010	0001010110100100
0000011100101100	0000101100100110	0010010101001001

2.4. $A(18,4,5) \geq 544$. We show that $A(18,4,5) \geq 544$ by using a cyclic group of order 17 that fixes the first coordinate. The 32 base blocks are:

001111000000000001	011001000000010100	010100101000001000
011101010000000000	011000110001000000	111100000010000000
011100100000100000	011000101000000100	111010000000000100
011100001100000000	011000100110000000	111000100000000010
011100000001001000	011000010010000010	111000010000001000
011010010000100000	011000001000010010	111000001001000000
011010001000001000	011000000101000010	110101000000010000
011010000100010000	011000000100100100	110100100000000100
011001001010000000	011000000010101000	110100010000100000
011001000100001000	010101001000000100	110010000010001000
011001000000100010	010101000000101000	

2.5. $A(21,4,5) \geq 1113$. We show that $A(21,4,5) \geq 1113$ by using a group of order 63 that acts on positions 0–20, generated by the three permutations

(0,2,1,4,5,3,6)(7,9,8,11,12,10,13)(14,16,15,18,19,17,20),
(0,2,4)(1,3,5)(7,9,11)(8,10,12)(14,16,18)(15,17,19),
(0,7,14)(1,8,15)(2,9,16)(3,10,17)(4,11,18)(5,12,19)(6,13,20).

The 19 base blocks are:

000000000011110000010	000000000001111010000	000000100011001001000
000000000000010111010	000000000011010010001	000000100001100000101
000000000111100000100	000000000101010000011	000000100101000100100
000000100000100110010	000000100010100001010	000000100101000010010
000000100010000111000	000000100110000001001	000000100010100100100
000000100000010001011	000000100011000010100	000000100010101010000
000000000000110101100		

3. The partitioning construction

We use the description and notation from [11] and [3], §VI.

A *partition* $\Pi(n,w) = (C_1, ..., C_m)$ is a partition of the set of all $\binom{n}{w}$ binary vectors of length n and weight w into codes C_i that all have minimum distance at least 4. By definition, $C_j = \emptyset$ for $j > m$. The vector $\pi(n,w) = (|C_1|, ..., |C_m|)$ is the *index vector* of the partition $\Pi(n,w)$.

The *direct product* $\Pi(n_1,w_1) \times \Pi(n_2,w_2)$ of two partitions $(C_1, ..., C_{m_1})$ and $(D_1, ..., D_{m_2})$ is the code $\bigcup_i C_i * D_i$ (of word length n_1+n_2 and weight w_1+w_2 and size $\sum |C_i|.|D_i|$), where for two codes C and D the code $C*D$ is the code consisting of all possible concatenations $c*d$ with $c \in C$ and $d \in D$.

The *partitioning construction* for codes of length n, weight w and minimum distance 4 constructs the code $C = \bigcup_i \Pi(n_1, 2i+\epsilon) \times \Pi(n_2, w-2i-\epsilon)$ where $n = n_1+n_2$ and $\epsilon \in \{0,1\}$ and the union is over all i with $i \geq 0$ and $2i+\epsilon \leq w$.

It is usually nontrivial to construct the required ingredients $\Pi(n,w)$. However, for $w \leq 1$ the partition is trivial, namely the partition into singletons, and for $w = 2$ the optimal partition is that of the $n(n-1)/2$ pairs into $n-1$ codes of size $n/2$ if n is even, and into n codes of size $(n-1)/2$ if n is odd. Partitions $\Pi(n,w)$ and $\Pi(n,n-w)$ are related by complementation. It is always possible to find a $\Pi(n,w)$ with at most n parts, cf. [12].

Example 1. We show that $A(18,4,7) \geq 2042$. Take $n_1 = 8$, $n_2 = 10$, $\epsilon = 1$, using direct products $\Pi(8,1) \times \Pi(10,6)$, $\Pi(8,3) \times \Pi(10,4)$, $\Pi(8,5) \times \Pi(10,2)$, $\Pi(8,7) \times \Pi(10,0)$. From a $\Pi(10,4)$ with sizes $(30,30,30,28,26,23,22,20,1)$ we find $\binom{10}{4} - 1 = 209$ for the first product, from $\Pi(8,3)$ with 7 codes of size 8 and a

$\Pi(10,6)$ with sizes $(30,30,30,30,30,22,22,12,2,2)$ we find $8(\binom{10}{4}-16)=1552$ for the second, then $5\binom{8}{5}=280$ for the third, and 1 for the last, 2042 altogether.

3.1. IMPROVEMENTS BY ETZION AND BITAN. The code C that results from the partitioning construction is not always maximal. Etzion and Bitan [9] gave a handful of examples of improvements. Let us redo two of their examples here (using improved ingredients).

Example 2. We show that $A(21,4,7) \geq 6161$. Take $n_1 = 10$, $n_2 = 11$, $\epsilon = 0$. The products $\Pi(10,0)\times\Pi(11,7)$, $\Pi(10,2)\times\Pi(11,5)$, $\Pi(10,4)\times\Pi(11,3)$, $\Pi(10,6)\times\Pi(11,1)$ contribute $A(11,4,4)+5\binom{11}{5}+17\binom{10}{4}+\binom{10}{4}=6125$. For $\Pi(11,5)$ and $\Pi(10,4)$ we used partitions with 9 codes, for $\Pi(11,3)$ the Etzion-Bitan partition with 9 codes of size 17, 1 code of size 3, and 9 codes of size 1, where the 9 codes of size 1 are the triples covering the pair $0^9 1^2$. Only the first 9 codes were used, and of $\Pi(11,1)$ also only the first 9 codes were used, so that the vector $0^9 1^2$ has distance at least 3 to all second halves used so far, and $A(10,4,5)=36$ vectors $u * 0^9 1^2$ can be added.

Example 3. We show that $A(21,4,8) \geq 10767$. Take $n_1 = 10$, $n_2 = 11$, $\epsilon = 1$. The products $\Pi(10,1)\times\Pi(11,7)$, $\Pi(10,3)\times\Pi(11,5)$, $\Pi(10,5)\times\Pi(11,3)$, $\Pi(10,7)\times\Pi(11,1)$ contribute $\binom{11}{4}+13\binom{11}{5}+17\binom{10}{5}+9.13=10737$. For $\Pi(11,7)$ we used a partition with 10 codes. For $\Pi(10,7)$ one with 9 codes of size 13 and a code of size 3, that is not used to leave room for $A(10,4,6)=30$ vectors $u * 0^9 1^2$.

3.2. VARYING THE SPLIT. Instead of keeping n_1 and n_2 fixed in the partitioning construction, one can use a varying split. For example, one can show that $A(23,4,8) \geq 23467$ using the union of $\Pi(12,0)\times\Pi(11,8)$, $\Pi(12,2)\times\Pi(11,6)$, $\Pi(12,4)\times\Pi(11,4)$, $\Pi(12,6)\times\{0\}\times\Pi(10,2)$, $\Pi(13,8)\times\Pi(10,0)$. Because $\Pi(12,6)$ can be taken to have 9 codes, nothing is lost by taking $\Pi(10,2)$ instead of $\Pi(11,2)$, but something is gained taking $\Pi(13,8)$ instead of $\Pi(12,8)$.

3.3. PARTITIONS USED. As we mentioned, partitions $\Pi(n,0)$ and $\Pi(n,1)$ are trivial, and it is easy to see what the best partitions $\Pi(n,2)$ are (cf. [3]). Nowadays also optimal partitions $\Pi(n,3)$ are known: If $n \equiv 1,3 \pmod 6$, $n \neq 7$, then a partition of all triples on n points into Steiner triple systems exists [15, 19, 13], so that we have a $\Pi(n,3)$ consisting of $n-2$ codes, each of size $n(n-1)/6$. Shortening these we find that for $n \equiv 0,2 \pmod 6$, $n \neq 6$, there is a partition $\Pi(n,3)$ consisting of $n-1$ codes, each of size $n(n-2)/6$. In [6, 7] partitions $\Pi(n,3)$ are constructed for $n \equiv 4 \pmod 6$, consisting of n codes, $n-1$ of size $(n^2-2n-2)/6$ and 1 of size $(n-1)/3$. Finally, [14] constructs partitions $\Pi(n,3)$ for $n \equiv 5 \pmod 6$, $n \neq 5$, with $n-1$ codes, $n-2$ of size $(n^2-n-8)/6$ and 1 of size $4(n-2)/3$. All of these are optimal.

To obtain the codes implied by Table 3 we used only the obvious partitions: for $w \leq 3$ the ones mentioned above, for $w=4$ the Graham-Sloane partitions with n codes ([3], Theorem 14), and finally for $w=5$ the partition with one code as large as possible (the best lower bound known for $A(n,4,w)$) and all other codes arbitrary, for example of size 1.

For Table 2 we spent some effort to find good partitions. In Table 1 we give the index vectors for the partitions used. The actual partitions can be found near [1].

There are three interesting partitions used in the next sections which will be given now. We give a partition $\Pi(11,4)$ with index vector $\pi(11,4)=(35,35,35,$

Table 1. Partitions used

n	w	#	part sizes														
8	4	6	14	14	12	12	10	8									
9	4	8	18	18	18	18	16	15	15	8							
9	4	9	18	18	18	17	17	17	13	7	1						
10	4	10	30	30	30	30	30	22	22	12	2	2					
10	4	9	30	30	30	28	26	23	22	20	1						
10	4	9	30	30	30	30	26	25	22	15	2						
10	5	8	36	36	34	34	29	29	27	27							
11	4	10	35	35	35	35	35	34	31	31	31	28					
11	4	10	35	35	35	35	35	35	31	30	30	29					
11	4	12	35	35	35	35	35	35	35	32	29	16	7	1			
11	4	13	35	35	35	34	34	34	34	34	34	12	3	3	3		
11	5	9	66	66	60	55	55	55	54	39	12						
12	4	11	51	51	51	51	49	47	47	46	45	43	14				
12	4	11	51	51	51	51	51	49	47	47	45	34	18				
12	4	11	51	51	51	51	51	50	50	47	41	27	25				
12	4	12	51	51	51	51	50	49	48	46	44	39	11	4			
12	4	12	51	51	51	51	51	50	50	44	41	38	16	1			
12	4	12	51	51	51	51	51	50	50	47	45	24	19	5			
12	4	13	51	51	51	51	51	50	50	46	46	26	13	8	1		
12	5	11	72	72	72	72	72	72	72	72	72	72	72				
12	5	13	80	80	80	80	76	72	70	68	67	62	45	11	1		
12	5	13	80	80	80	80	77	73	73	71	64	50	45	18	1		
12	6	9	132	132	120	110	110	110	108	78	24						
13	4	13	65	65	65	65	65	60	59	58	58	56	55	33	11		
13	4	13	65	65	65	65	65	61	60	59	58	55	49	42	6		
13	4	13	65	65	65	65	65	62	60	59	58	56	50	32	13		
13	4	13	65	65	65	65	65	62	61	61	57	49	47	43	10		
13	5	13	123	123	123	122	114	111	107	105	98	95	89	53	24		
13	5	13	123	123	123	122	114	112	109	102	95	92	84	76	12		
13	5	13	123	123	123	122	114	112	109	102	97	92	85	72	13		
13	5	13	123	123	123	122	114	113	111	104	100	89	79	62	24		
13	5	14	123	123	122	122	114	111	107	107	101	95	85	56	19	2	
13	6	13	166	166	166	159	151	146	139	136	123	120	111	98	35		
13	6	13	166	166	166	159	151	149	138	136	127	118	106	98	36		
13	6	13	166	166	166	159	151	149	138	136	130	119	103	95	38		
13	6	13	166	166	166	159	151	149	141	136	130	115	101	95	41		
13	6	14	166	166	165	159	151	149	138	136	130	121	104	95	34	2	
13	6	14	166	166	166	157	151	149	140	134	131	117	115	82	41	1	
13	6	14	166	166	166	159	151	149	139	135	129	114	106	101	34	1	
13	6	14	166	166	166	159	151	149	141	134	130	118	111	82	41	2	
14	4	13	91	91	91	90	88	82	81	77	77	73	68	64	28		
14	4	14	91	91	91	90	88	82	79	78	78	73	68	65	26	1	
14	4	14	91	91	91	90	88	82	80	79	78	72	67	65	25	2	
14	4	14	91	91	91	90	88	82	81	78	76	71	69	66	26	1	
14	4	14	91	91	91	90	88	82	81	78	78	73	70	57	28	3	
14	4	14	91	91	91	90	88	82	81	78	78	74	69	55	30	3	
14	5	15	169	169	169	169	157	156	151	148	145	143	132	123	106	57	8
14	5	15	169	169	169	169	157	157	149	148	144	143	131	125	106	62	4
14	5	15	169	169	169	169	157	157	149	149	143	142	132	124	108	60	5
14	6	14	253	252	247	247	242	236	212	212	212	212	212	212	212	42	
14	6	14	278	276	272	259	247	229	227	220	209	204	186	176	150	70	
14	6	15	278	275	272	258	248	228	227	218	213	206	195	167	139	77	2
14	6	15	278	276	272	259	247	229	227	220	210	207	186	180	139	68	5
14	6	15	278	276	272	259	247	229	227	220	212	206	186	180	135	71	5
14	6	15	278	276	272	259	248	229	226	218	209	205	186	178	153	64	2
14	6	15	278	276	273	258	248	231	229	219	208	207	189	168	138	77	4
14	6	15	278	276	273	259	248	231	228	222	209	199	185	171	144	79	1
14	7	14	282	282	281	281	276	276	274	274	268	268	265	265	126	14	
14	7	15	325	325	317	304	282	268	262	246	238	221	206	195	160	80	3

$35, 33, 32, 32, 32, 31, 30)$ explicitly (in the notation of [3], but with 0 instead of A for the tenth color). The words are ordered lexicographically and we list for each word its color (each color represents a different code of the partition).

41352963146520839858201477160295143712094375680253437916544830219257686793579681
24719583274025169730308460153672184946807392515869687914024457183627136249659308
73960215408615921087341302480253761098549124825736857679489315398002740472313586
84259239648185240176916073321024406565931165827870932440210523173762568940175439
8259768431

We also give a partition $\Pi(11, 5)$ with index vector $\pi(11, 5) = (66, 66, 60, 55, 55, 55, 54, 39, 12)$. No code has two disjoint 5-sets, so extending by a point and adding complements yields a partition $\Pi(12, 6)$ with the index vector $\pi(12, 6) = (132, 132, 120, 110, 110, 110, 108, 78, 24)$.

26371558136437162752431567712495248367424752312138653167168742437541236728331247
51566527431465828331236572116755211479846478328745315422643172617541782795243138
65271328164436638751218439662355821493261356216731858724468514267332175812453546
72149373521675341825317664219242638575321764813241683472564135723813576131624749
25356242716371491832576453815182237461792346518582648265314135612276327761593887
6428345161452763582417312546251783251847734545621172698368127

3.4. TABLES WITH LOWER BOUNDS FROM PARTITIONING. Table 2 below gives lower bounds on $A(n, d, w)$, where $d = 4$ and $2w \leq n$. These lower bounds are obtained using the partitioning construction.

TABLE 2. Lower bounds for $A(n, 4, w)$

$n \backslash w$	6	7	8	9	10	11	12	13	14
18	1260^p	2042	3186^s	3540^s					
19	1620^n	3172^p	4667^p	6726^s					
20	2304^n	4213^p	7730^p	10048	13452^g				
21	2856^s	6161	10767	17177	20654				
22	3927^s	8338	16527	25902	37127	40624			
23	$5313.^s$	11696	23467	41413	58659	76233			
24	$7084.^S$	15656^{eb}	34914^g	59904	98852	118422	151484		
25	7787	21220	47265	89742	142373	198387	231530		
26	10010^p	27050	66352	129708	222775	320584	401937	431724	
27	12012^s	35874	88604	188561	334859	518014	686164	791461	
28	15288^g	44915	122685	263008	508952	819041	1167909	1420920	1535756

Legenda: ^{s}: shortened code. ^{S}: Steiner system $S(5, 6, 24)$. ^{g}: a group code from [3]. ^{n}: idem from [17]. ^{p}: product construction from [3]. ^{eb}: idem from [9]. Unmarked entries are from this paper.

In Table 3 below we give lower bounds for $A(n, 4, 5)$ for $29 \leq n \leq 64$, to be compared with the table in [18]. It improves all bounds from that paper except for the three values for $n = 45, 46, 47$. The values marked ^{s} are derived from Steiner systems $S(5, 6, 36)$ and $S(5, 6, 48)$ [2, 5]. The values marked with a dot are exact.

4. THE CHROMATIC NUMBERS OF THE JOHNSON GRAPHS

The Johnson graph $J(n, w)$ is the graph on the binary vectors of length n and weight w, adjacent when they have Hamming distance 2. The graphs $J(n, w)$ and $J(n, n - w)$ are isomorphic. The chromatic number $\chi = \chi(n, w) = \chi(J(n, w))$ is the smallest number of distance-at-least-4 codes its vertex set can be partitioned

Table 3. Lower bounds for $A(n,4,5)$

n	29	30	31	32	33	34	35	36	37
bd	4423	4901	5697	6582	7656^s	$8976.^s$	$10472.^s$	10948	12473
n	38	39	40	41	42	43	44	45	46
bd	13471	15010	17119	19258	20671	22728	25564	28413^s	$31878.^s$
n	47	48	49	50	51	52	53	54	55
bd	$35673.^s$	36809	40560	42920	46612	51420	56251	59293	63973
n	56	57	58	59	60	61	62	63	64
bd	69931	75550	79330	85728	93206	100527	105472	112457	121902

into. It is easy to verify that $\chi(J(n,w))$ is the length of the shortest index vector $\pi(n,w)$. One has $\max(w+1, n-w+1) \leq \chi(n,w) \leq n$, where the lower bound is the size of a maximal clique, and the upper bound is due to [12]. One also has the monotonicity inequalities $\chi(n,w) \geq \chi(n-1,w-1)$ and $\chi(n,w) \geq \chi(n-1,w)$ as an immediate consequence of shortening codes in a partition. Table 4 below gives lower and upper bounds for χ.

Table 4. Bounds on the chromatic number of Johnson graphs

$n \backslash w$	1	2	3	4	5	6	7	8
5	5	5						
6	6	5	6					
7	7	7	6					
8	8	7	7	6				
9	9	9	7	8				
10	10	9	10	8–9	8			
11	11	11	10	10	8–9			
12	12	11	11	10–11	10–11	8–9		
13	13	13	11	11–13	10–13	10–13		
14	14	13	13	11–13	11–14	10–14	10–14	
15	15	15	13	13–14	12–15	11–15	10–15	
16	16	15	16	13–14	13–15	12–15	11–16	10–15

Discussion. The cases $w = 1$ and $w = 2$ are trivial. For $w \geq 3$ and $n \leq 14$ the upper bounds were already given in [3] (Table VI), except that $\chi(11,4) \leq 10$ and $\chi(11,5) \leq \chi(12,6) \leq 9$ were given above, and $\chi(12,5) \leq 11$ follows from a partition $\Pi(12,5)$ with 11 codes of size 72 given in [9]. That $\chi(15,3) = 13$ follows from the existence of a large set of STS(15) [4]. Optimal partitions $\Pi(11,3)$ with parts 17^9 12 (9 codes of size 17 and one codes of size 12) and $\Pi(17,3)$ with parts 44^{15} 20 were given in [14]. An optimal partition $\Pi(16,3)$ with parts 37^{15} 5 was constructed by Doron Cohen and given in [6]. In particular, $\chi(16,3) = 16$, in spite of the claim in [9] that $\chi(16,w) \leq 15$ for $2 \leq w \leq 6$. A partition $\Pi(16,4)$ with codes 140^6 136^6 116 48 was given in [8]. That $\Pi(16,w) \leq 15$ for $w = 6, 8$ follows from [11]. A partition $\Pi(16,5)$ with codes 302^{14} 140 is found by the product construction from [11], since one can take the union of the last two codes of size 70 each.

Concerning the lower bounds for $w \geq 4$, all except two follow from monotonicity. We verified explicitly that $\chi(9,4) > 7$—there are five, not seven mutually disjoint codes of word length $n = 9$, constant weight $w = 4$ and size 18. That $\chi(15,5) > 11$ follows since there is no Steiner system $S(4,5,15)$ [16], let alone eleven mutually disjoint ones.

In the case of $J(10,4)$ there exists a coloring with 9 colors where the last color is used only once. So $J(10,4)$ minus a vertex has chromatic number 8.

It would be interesting to give more general constructions for colorings of $J(n,w)$ with fewer than n colors.

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